Module 1 Lesson 4
Systems of Linear Equations in Two Variables and Applications

Objectives

In this lesson, we will learn how to

- check whether an ordered pair solves a system of equations;
- solve a system by substitution;
- solve a system by elimination (addition);
- solve a system by graphing;
- classify systems as consistent/inconsistent and dependent/independent;
- apply systems of equations to real situations.

1. Systems of Equations

A system of equations is two or more equations considered together, for example

$$\begin{cases} 5x - 4y = 20, \\ 2x - 3y = 1. \end{cases}$$

The solution to a system is the ordered pair that makes both equations true. A system may also have infinitely many solutions or no solution at all.

Example 1: Determine whether $(8, 5)$ is a solution to the system above.

Solution

Substitute $x = 8$, $y = 5$ into both equations.

$$5(8) - 4(5) = 40 - 20 = 20. \checkmark$$

$$2(8) - 3(5) = 16 - 15 = 1. \checkmark$$

Both equations are true, so $(8, 5)$ is a solution to the system.

2. Solving by Substitution

Substitution Method

Solve one equation for one variable, then substitute that expression into the other equation. This produces a single equation in one variable, which can be solved.

Example 2: Solve using the substitution method.

(1) $\begin{cases} x - y = 5, \\ 2x - 5y = 1. \end{cases}$

(2) $\begin{cases} 3x - y = -1, \\ 2x + y = 11. \end{cases}$

Solution

(1) From the first equation, $x = y + 5$. Substitute into the second:

$$2(y + 5) - 5y = 1.$$

$$2y + 10 - 5y = 1.$$

$$-3y = -9 \implies y = 3.$$

Then $x = 3 + 5 = 8$. The solution is $(8, 3)$.
(2) From the first equation, $y = 3x + 1$. Substitute into the second:

$$2x + (3x + 1) = 11.$$

$$5x + 1 = 11 \implies x = 2.$$

Then $y = 3(2) + 1 = 7$. The solution is $(2, 7)$.

3. Solving by Elimination (Addition)

Elimination Method

Combine the equations, possibly after multiplying one or both by a constant, so that one variable cancels when the equations are added.

Example 3: Solve using the elimination method.

(1) $\begin{cases} 3x - 2y = 5, \\ x + y = 5. \end{cases}$

(2) $\begin{cases} 2x + 6y = 7, \\ 3x + 9y = 10. \end{cases}$

Solution

(1) Multiply the second equation by 2:

$$2x + 2y = 10.$$

Add to the first equation:

$$(3x - 2y) + (2x + 2y) = 5 + 10 \implies 5x = 15 \implies x = 3.$$

From $x + y = 5$: $y = 5 - 3 = 2$. The solution is $(3, 2)$.
(2) Multiply the first equation by 3 and the second by 2:

$$6x + 18y = 21,$$

$$6x + 18y = 20.$$

Subtracting gives

$$0 = 1,$$

which is false. This system has no solution --- the two lines are parallel.

4. Applications

Example 4: A small business shipped 120 packages one day. Customers are charged $\$3.50$ for each standard delivery package and $\$7.50$ for each express delivery package. Total shipping charges for the day were $\$596$. How many of each kind of package were shipped?

Solution

Let x be the number of standard packages and y the number of express packages.

$$x + y = 120,$$

$$3.5x + 7.5y = 596.$$

From the first equation, $x = 120 - y$. Substitute:

$$3.5(120 - y) + 7.5y = 596.$$

$$420 - 3.5y + 7.5y = 596.$$

$$420 + 4y = 596 \implies 4y = 176 \implies y = 44.$$

Then $x = 120 - 44 = 76$.
There were 76 standard packages and 44 express packages.

Example 5: A bus company counted 32 ticket receipts last Friday. A senior ticket costs $\$6.75$ and a regular ticket costs $\$10$. The driver collected $\$297.25$ total. Let x be the number of senior tickets and y the number of regular tickets. Find the ordered pair (x, y).

Solution

$$x + y = 32,$$

$$6.75x + 10y = 297.25.$$

From the first equation, $y = 32 - x$. Substitute:

$$6.75x + 10(32 - x) = 297.25.$$

$$6.75x + 320 - 10x = 297.25.$$

$$-3.25x = -22.75 \implies x = 7.$$

Then $y = 32 - 7 = 25$. The ordered pair is $(7, 25)$.

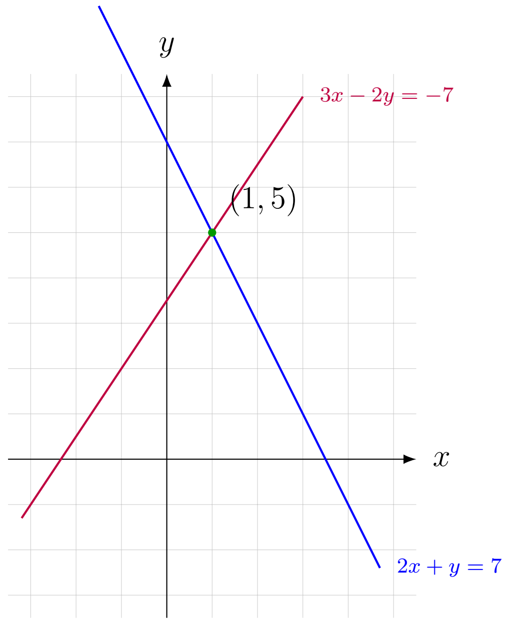
5. The Graphical Method

Graphical Method

To solve a system by graphing, draw both lines and find the point(s) where they cross.

Example 6: Solve by graphing:

$$\begin{cases} 3x - 2y = -7, \\ 2x + y = 7. \end{cases}$$



Solution

Graph the first equation. For the x-intercept, set $y = 0$: $3x = -7 \Rightarrow x = -\frac{7}{3}$. For the y-intercept, set $x = 0$: $-2y = -7 \Rightarrow y = \frac{7}{2}$.
Graph the second equation. For the x-intercept, set $y = 0$: $2x = 7 \Rightarrow x = \frac{7}{2}$. For the y-intercept, set $x = 0$: $y = 7$.
Graphing both lines shows they intersect at

$$(1, 5).$$

Check:

$$3(1) - 2(5) = 3 - 10 = -7. \checkmark$$

$$2(1) + 5 = 2 + 5 = 7. \checkmark$$

6. Classifying Systems

Consistent or Inconsistent

A system is consistent if it has one or more solutions --- the lines intersect or coincide. A system is inconsistent if it has no solution --- the lines are parallel.

Dependent or Independent

If a system represents the same line, every point on that line satisfies both equations, so there are infinitely many solutions; we call the equations dependent. If the equations represent two different lines, we call them independent.

Summary

Key Ideas

- A system's solution is the ordered pair satisfying every equation in the system.
- Substitution: solve for one variable, then substitute into the other equation.
- Elimination: multiply and add/subtract equations to cancel a variable.
- Graphing: the solution is the intersection point of the two lines.
- A false statement (like $0 = 1$) after elimination means no solution (inconsistent, parallel lines).
- A true statement (like $0 = 0$) after elimination means infinitely many solutions (dependent, same line).