Module 1 Lesson 5
Linear Inequalities

Objectives

In this lesson, we will learn how to

- apply the addition and multiplication principles for inequalities;
- solve linear and double inequalities;
- express solutions in interval notation and on a number line;
- apply inequalities to real-world problems.

1. Principles for Solving Inequalities

An inequality is a mathematical sentence containing $<$, $>$, $\leq$, or $\geq$. The solution is all values of the variable that make the inequality true.

Addition Principle

For any real numbers a, b, c: if $a < b$ is true, then $a + c < b + c$ is true.

Multiplication Principle

(a) If $a < b$ and $c > 0$, then $ac < bc$.

(b) If $a < b$ and $c < 0$, then $ac > bc$.

Important Note

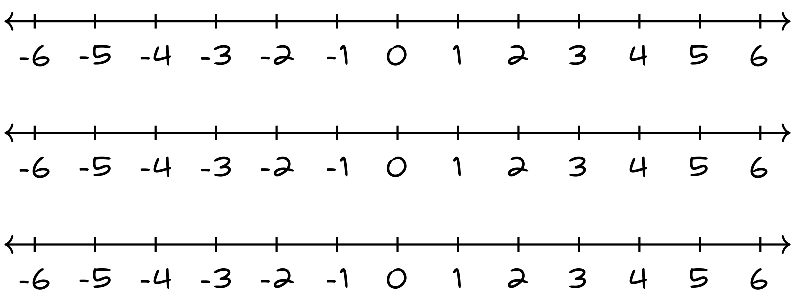
When both sides of an inequality are multiplied (or divided) by a negative number, the inequality sign must be reversed.

Example 1: Solve each inequality. Express the solution in interval notation, and graph the solution.

(a) $2x - 1 > x + 5$

(b) $3 - 2x < x + 9$

(c) $-\frac{1}{2}x \geq 2 + \frac{2}{3}x$



Solution

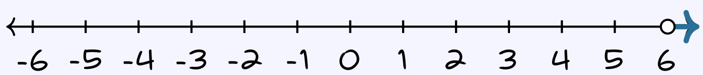
(a)

$$2x - 1 > x + 5.$$

$$2x - x > 5 + 1.$$

$$x > 6.$$

Interval notation: $(6, \infty)$.



(b)

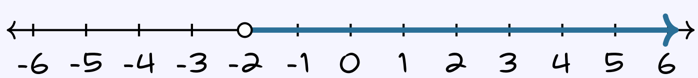
$$3 - 2x < x + 9.$$

$$3 - 9 < x + 2x.$$

$$-6 < 3x.$$

$$x > -2.$$

Interval notation: $(-2, \infty)$.



(c) Multiply both sides by 6 to clear fractions:

$$6\left(-\frac{1}{2}x\right) \geq 6(2) + 6\left(\frac{2}{3}x\right).$$

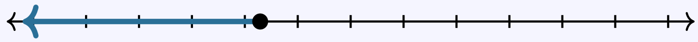
$$-3x \geq 12 + 4x.$$

$$-7x \geq 12.$$

Dividing by -7 reverses the inequality:

$$x \leq -\frac{12}{7}.$$

Interval notation: $\left(-\infty, -\frac{12}{7}\right]$.

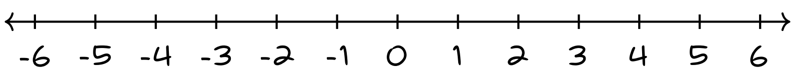


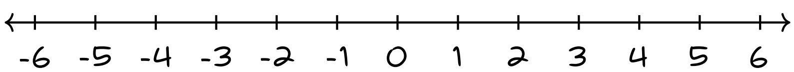
2. Double Inequalities

Example 2: Solve each double inequality. Express the solution in interval notation and graph the solution.

(a) $3 < 5 + 4x < 6$

(b) $2x - 3 \leq 4x + 3 < 2x + 5$





Solution

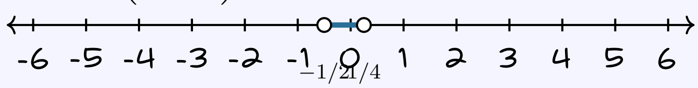
(a) Subtract 5 from all three parts:

$$-2 < 4x < 1.$$

Divide by 4:

$$-\frac{1}{2} < x < \frac{1}{4}.$$

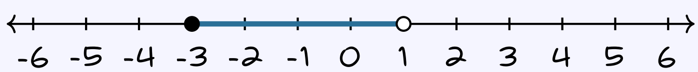
Interval notation: $\left(-\frac{1}{2}, \frac{1}{4}\right)$.



(b) Split into two inequalities.
Left: $2x - 3 \leq 4x + 3 \implies -6 \leq 2x \implies x \geq -3$.
Right: $4x + 3 < 2x + 5 \implies 2x < 2 \implies x < 1$.
Combining:

$$-3 \leq x < 1.$$

Interval notation: $[-3, 1)$.



3. Verbal Problems and Applications

Example 3:

(a) If 55 more than x is bigger than -124, determine all possible values of x.

(b) Ethan has $\$90$ to spend on juice boxes. Each juice box costs $\$2.51$. What is the maximum number of juice boxes he can buy?

(c) Saanvi can be paid in one of two ways for painting a house: Plan A pays $\$200$ plus $\$12$ per hour; Plan B pays $\$20$ per hour. If a job takes n hours, for what values of n is Plan A better financially?

Solution

(a)

$$x + 55 > -124.$$

$$x > -179.$$

Interval notation: $(-179, \infty)$.
(b) We need

$$2.51n \leq 90.$$

$$n \leq \frac{90}{2.51} \approx 35.86.$$

Since n must be a whole number, the maximum is $n = 35$ juice boxes.
(c) Plan A pays $200 + 12n$; Plan B pays $20n$. Plan A is better when

$$200 + 12n > 20n.$$

$$200 > 8n.$$

$$n < 25.$$

Plan A is the better financial choice when the job takes fewer than 25 hours.

Summary

Key Ideas

- Adding or subtracting the same number from both sides of an inequality never changes the direction of the inequality.
- Multiplying or dividing both sides by a negative number reverses the inequality sign.
- Use an open circle for $<$ or $>$, and a closed circle for $\leq$ or $\geq$.
- Double inequalities can be solved all at once (if the variable appears the same way in all parts) or split into two separate inequalities and combined.
- Solutions are typically expressed using interval notation and graphed on a number line.