Module 3 Lesson 2
Graphs of Polynomial Functions

Objectives

In this lesson, we will learn how to

- sketch the graph of a polynomial function using end behavior, intercepts, and multiplicity;
- read a polynomial's graph to recover its zeros, multiplicities, and a possible formula.

1. Strategy for Sketching a Polynomial

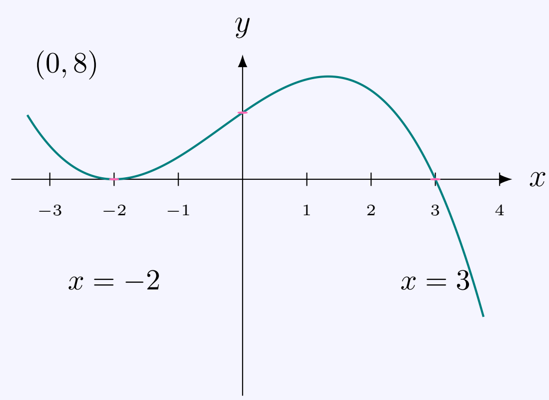
Steps to Sketch $f(x)$

1. Determine the end behavior (degree and sign of leading coefficient).
2. Find the y-intercept: evaluate $f(0)$.
3. Find the zeros (factor if necessary) and their multiplicities.
4. Determine whether the graph crosses or bounces at each zero.
5. Plot a few additional points if needed, then connect with a smooth, continuous curve consistent with the end behavior.

Example 1: Sketch the graph of $f(x) = -\frac{2}{3}(x+2)^2(x-3)$.

Solution

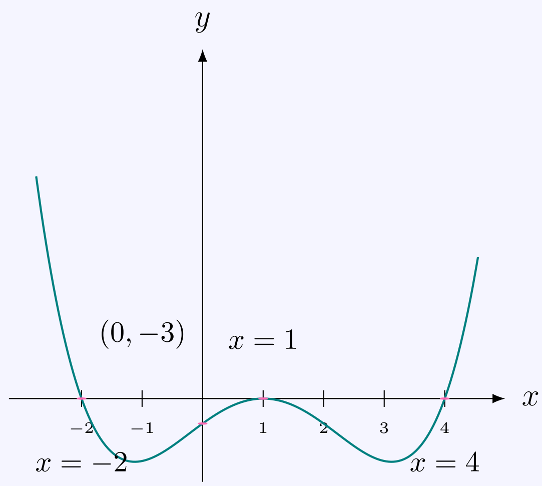
End behavior: degree 3 (odd), leading coefficient $-\frac{2}{3} < 0$: left up, right down.
y-intercept: $f(0) = -\frac{2}{3}(2)^2(-3) = -\frac{2}{3}(4)(-3) = 8$, i.e. $(0, 8)$.
Zeros: $x = -2$ (multiplicity 2, bounce), $x = 3$ (multiplicity 1, cross).



Example 2: Sketch the graph of $f(x) = \dfrac{3}{8}(x+2)(x-1)^2(x-4)$.

Solution

End behavior: degree 4 (even), leading coefficient $\frac{3}{8} > 0$: both ends go up.
y-intercept: $f(0) = \frac{3}{8}(2)(1)(-4) = \frac{3}{8}(-8) = -3$, i.e. $(0, -3)$.
Zeros: $x = -2$ (multiplicity 1, cross), $x = 1$ (multiplicity 2, bounce), $x = 4$ (multiplicity 1, cross).

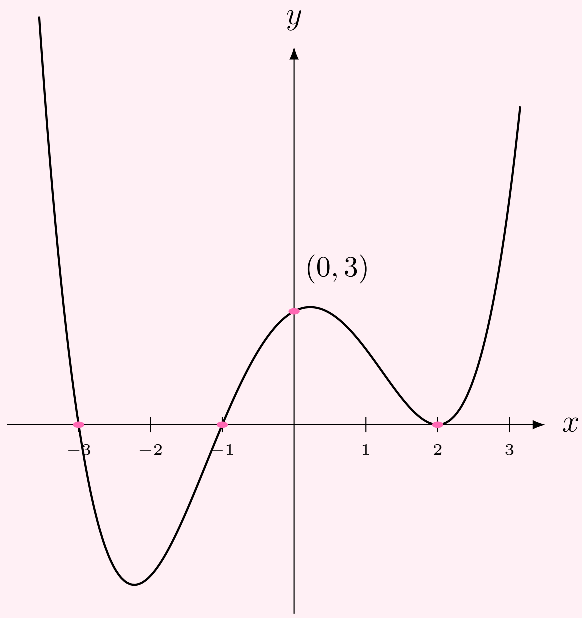


2. Reading a Graph

Reading Zeros and Multiplicity from a Graph

- Where the graph crosses the x-axis, the corresponding zero has odd multiplicity (often 1).
- Where the graph bounces (touches and turns), the corresponding zero has even multiplicity (often 2).
- Multiply the leading coefficient's sign and the end behavior together with the zeros to build a possible formula $f(x) = a(x - r_1)^{m_1}(x - r_2)^{m_2} \cdots$.

Example 3: The graph of a degree-4 polynomial is shown below. Determine its zeros, their multiplicities, and a possible formula for $f(x)$.



Solution

The graph crosses at $x = -3$ and $x = -1$ (odd multiplicity, take 1), and bounces at $x = 2$ (even multiplicity, take 2). This already accounts for degree $1 + 1 + 2 = 4$, matching the given degree.
A possible formula:

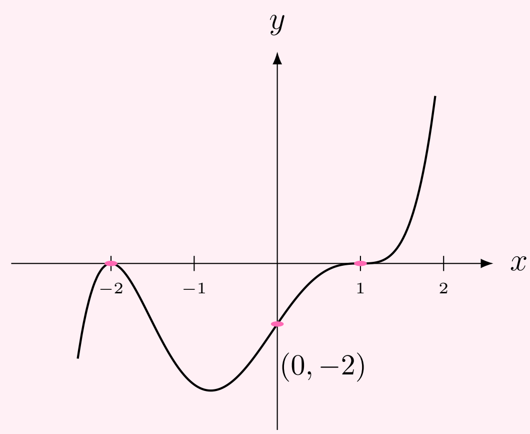
$$f(x) = a(x + 3)(x + 1)(x - 2)^2.$$

Use the y-intercept $(0, 3)$ to find a:

$$3 = a(3)(1)(4) = 12a \implies a = \frac{1}{4}.$$

$$\boxed{f(x) = \frac{1}{4}(x + 3)(x + 1)(x - 2)^2}$$

Example 4: The graph of a degree-5 polynomial is shown below. Determine its zeros, their multiplicities, and a possible formula for $f(x)$.



Solution

The graph bounces at $x = -2$ (even multiplicity, take 2) and crosses with a flattened S-shape at $x = 1$ (odd multiplicity greater than 1, take 3). This accounts for degree $2+3=5$, matching the given degree. A possible formula:

$$f(x) = a(x+2)^2(x-1)^3.$$

Use the y-intercept $(0,-2)$ to find a:

$$-2 = a(4)(-1) = -4a \implies a = \frac{1}{2}.$$

$$\boxed{f(x) = \frac{1}{2}(x+2)^2(x-1)^3}$$

Summary

Key Ideas

- To sketch a polynomial: find the end behavior, the y-intercept, the zeros and their multiplicities, whether the graph crosses or bounces at each zero, then connect with a smooth curve.
- To read a graph: crossings correspond to odd-multiplicity zeros (usually 1), and bounces correspond to even-multiplicity zeros (usually 2); the multiplicities should add up to the given degree.
- Once the zeros and their multiplicities are known, write $f(x) = a(x - r_1)^{m_1}(x - r_2)^{m_2} \dots$ and use a known point (often the y-intercept) to solve for a.