Module 3 Lesson 4
Radical Equations

Objectives

In this lesson, we will learn how to

- solve equations involving radicals;
- recognize and discard extraneous solutions;
- apply radical equations to real-world formulas.

1. Radical Equations

Radical Equation

A radical equation is an equation in which the variable appears inside a radical, such as $\sqrt[n]{ax+b}=c$. The radicand is the expression under the radical, and n is the index. Recall that $\sqrt[n]{x}=x^{1/n}$.

How to Solve a Radical Equation

1. Isolate a radical on one side of the equation.
2. Raise both sides to a power that matches the radical's index (square both sides for a square root, cube both sides for a cube root, etc.).
3. If a radical remains, repeat steps 1--2.
4. Solve the resulting equation.
5. Check every solution in the original equation.
6. Discard any extraneous solutions.

Extraneous Solution

An extraneous solution is a value obtained algebraically (typically after squaring both sides) that does not satisfy the original equation. Squaring both sides can introduce solutions that were not there originally, so every solution must be checked.

2. Solving Radical Equations

Example 1: Solve $\sqrt{5-x}-3=0$.

Solution

$$\sqrt{5-x}=3 \implies 5-x=9 \implies x=-4.$$

Check: $\sqrt{5-(-4)}-3=\sqrt{9}-3=3-3=0$. ✓

$$\boxed{x=-4}$$

Example 2: Solve $\sqrt{15-2x}=x$.

Solution

$$15-2x=x^2 \implies x^2+2x-15=0 \implies (x+5)(x-3)=0.$$

$$x=-5 \text{ or } x=3.$$

Check $x=-5$: $\sqrt{15-2(-5)}=\sqrt{25}=5$, but $x=-5$. Since $5\neq-5$, this is extraneous.
Check $x=3$: $\sqrt{15-2(3)}=\sqrt{9}=3=x$. ✓

$$\boxed{x=3}$$

Example 3: Solve $\sqrt{x^2-2}-\sqrt{x+4}=0$.

Solution

$$\sqrt{x^2-2}=\sqrt{x+4} \implies x^2-2=x+4 \implies x^2-x-6=0.$$

$$(x-3)(x+2)=0.$$

So $x=3$ or $x=-2$.
Check $x=3$: $\sqrt{9-2}=\sqrt{7}$ and $\sqrt{3+4}=\sqrt{7}$. ✓
Check $x=-2$: $\sqrt{4-2}=\sqrt{2}$ and $\sqrt{-2+4}=\sqrt{2}$. ✓

$$\boxed{x=3,\ x=-2}$$

Example 4: Solve $\sqrt{3t+1}+\sqrt{5-t}=4$.

Solution

Isolate one radical: $\sqrt{3t+1} = 4 - \sqrt{5-t}$. Square both sides:

$$3t + 1 = 16 - 8\sqrt{5-t} + (5-t) = 21 - t - 8\sqrt{5-t}.$$

$$3t + 1 - 21 + t = -8\sqrt{5-t} \implies 4t - 20 = -8\sqrt{5-t}.$$

$$t - 5 = -2\sqrt{5-t}.$$

Multiply by -1: $5 - t = 2\sqrt{5-t}$. Let $u = \sqrt{5-t}$, so $5 - t = u^2$:

$$u^2 = 2u \implies u^2 - 2u = 0 \implies u(u-2) = 0.$$

$$u = 0 \text{ or } u = 2.$$

If $u = 0$: $\sqrt{5-t} = 0 \implies t = 5$.
If $u = 2$: $\sqrt{5-t} = 2 \implies 5 - t = 4 \implies t = 1$.
Check $t = 5$: $\sqrt{16} + \sqrt{0} = 4 + 0 = 4$. ✓
Check $t = 1$: $\sqrt{4} + \sqrt{4} = 2 + 2 = 4$. ✓

$$\boxed{t = 5,\ t = 1}$$

Example 5: Solve $4 + \sqrt[3]{x-6} = 2$.

Solution

$$\sqrt[3]{x-6} = -2 \implies x - 6 = (-2)^3 = -8 \implies x = -2.$$

Check: $4 + \sqrt[3]{-2-6} = 4 + \sqrt[3]{-8} = 4 + (-2) = 2$. ✓

$$\boxed{x = -2}$$

3. An Application: Skid-Mark Speed

Skid-Mark Formula

The speed v (in ft/sec) of a car that leaves a skid mark of length d (in feet) on dry concrete satisfies

$$v = k\sqrt{d}, \qquad k \approx 1.5.$$

Example 6:

(a) A car leaves a 54-foot skid mark. Find its speed.

(b) A car braking at 110 ft/sec (75 mph) leaves a skid mark. Find the length of the skid mark.

Solution

(a)

$$v = 1.5\sqrt{54} = 1.5(3\sqrt{6}) = 4.5\sqrt{6} \approx 11.02 \text{ ft/sec}.$$

(b) Solve $110 = 1.5\sqrt{d}$ for d:

$$\sqrt{d} = \frac{110}{1.5} = \frac{220}{3} \implies d = \left(\frac{220}{3}\right)^2 = \frac{48400}{9} \approx 5377.8 \text{ ft}.$$

Summary

Key Ideas

- To solve a radical equation: isolate a radical, raise both sides to a power matching the index, repeat if another radical remains, then solve.
- Raising both sides to a power can introduce extraneous solutions --- always check every solution in the original equation and discard any that fail.
- Radical equations model real formulas, such as the skid-mark speed formula $v = k\sqrt{d}$.