Chapter 4 Probability
Basic Concepts of Probability (4.1)

Objectives

In this lesson, we will learn how to

- identify events, simple events, and sample spaces;
- use three common approaches to find probabilities;
- interpret probability values between 0 and 1;
- use complements to find probabilities;
- identify significantly high and significantly low results;
- distinguish actual odds from payoff odds.

1. Basic Probability Ideas

Important Vocabulary

- An event is any collection of results or outcomes of a procedure.
- A simple event is an outcome that cannot be broken down into simpler components.
- The sample space is the complete list of all possible simple events.

For example, if one baby is born, the sample space is

$$\{b, g\},$$

where b means boy and g means girl.

If three babies are born, the sample space is

$$\{bbb, bbg, bgb, bgg, gbb, gbg, ggb, ggg\}.$$

Example 1: For the procedure of three births, answer the following.

(a) Give the sample space.

(b) Is ``2 girls and 1 boy'' a simple event?

(c) Is ggb a simple event?

Solution

(a) The sample space is

$$\{bbb, bbg, bgb, bgg, gbb, gbg, ggb, ggg\}.$$

(b) No. The event ``2 girls and 1 boy'' can happen in several ways: ggb, gbg, bgg. Therefore, it is not a simple event.

(c) Yes. The outcome ggb is one specific outcome, so it is a simple event.

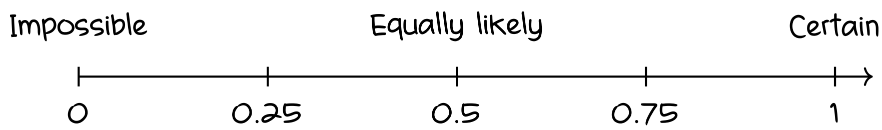
2. Probability Notation

Notation

- P denotes a probability.
- A, B, C usually denote events.
- $P(A)$ means the probability that event A occurs.

All probability values must be between 0 and 1:

$$0 \leq P(A) \leq 1.$$



How to Interpret Probability

- $P(A) = 0$: event A is impossible.
- $P(A) = 1$: event A is certain.
- A small probability, such as 0.001, describes an event that rarely occurs.

3. Three Common Approaches to Probability

Relative Frequency Approach

Conduct or observe a procedure many times and count how often event A occurs.

$$P(A) \approx \frac{\text{number of times } A \text{ occurred}}{\text{number of times the procedure was repeated}}.$$

Classical Approach

Use this approach only when all simple events are equally likely. If event A can occur in s ways among n equally likely simple events, then

$$P(A) = \frac{s}{n}.$$

Subjective Probability

Use knowledge of the relevant circumstances to estimate the probability.

Simulation

A simulation is a process that behaves like the actual procedure, so that similar results are produced. Simulations are useful when exact probability calculations are difficult.

4. Rounding Probabilities

When writing probabilities, give an exact fraction or decimal if possible. Otherwise, round final decimal answers to three significant digits.

Example 2: In a recent year, there were about 39 million commercial airline flights, and 16 of them crashed. Estimate the probability that a commercial airliner will crash on a given flight.

Solution

Use the relative frequency approach:

$$P(\text{crash}) \approx \frac{16}{39{,}000{,}000}.$$

$$P(\text{crash}) \approx 0.000000410.$$

This is a very small probability.

5. Law of Large Numbers

Law of Large Numbers

As a procedure is repeated again and again, the relative frequency probability of an event tends to approach the actual probability.

Caution

The law of large numbers describes behavior over many trials. It does not predict one individual outcome.

For example, if a fair coin is tossed many times, the proportion of heads should get closer to 0.5. But this does not mean the next toss must be heads or tails.

6. Relative Frequency Examples

Example 3: In a Pew Research Center survey, 366 adults answered ``yes'' when asked if they had seen or been in the presence of a ghost, and $1637$ adults answered ``no.'' Based on these results, find the probability that a randomly selected adult says ``yes.''

Solution

First find the total number of responses:

$$366 + 1637 = 2003.$$

Then

$$P(\text{yes}) = \frac{366}{2003} \approx 0.183.$$

There is about a 0.183 probability that a randomly selected adult says that they have seen or been in the presence of a ghost.

7. Complementary Events

Complement

The complement of event A, denoted by $\overline{A}$, consists of all outcomes in which event A does not occur.

$$P(\overline{A}) = 1 - P(A)$$

$$P(A) = 1 - P(\overline{A})$$

Example 4: In a survey of 2002 adults, 1782 said that they use the Internet. Find the probability that a randomly selected adult does not use the Internet.

Solution

The number who do not use the Internet is

$$2002 - 1782 = 220.$$

Therefore,

$$P(\text{does not use Internet}) = \frac{220}{2002} \approx 0.110.$$

8. Identifying Significant Results

Rare Event Rule for Inferential Statistics

If, under a given assumption, the probability of an observed event is very small, and the event occurs significantly less than or significantly greater than expected, we conclude that the assumption is probably not correct.

Significantly High or Low

- Significantly high: x successes is significantly high if

$$P(x \text{ or more}) \le 0.05.$$

- Significantly low: x successes is significantly low if

$$P(x \text{ or fewer}) \le 0.05.$$

The cutoff 0.05 is commonly used, but it is not absolutely rigid.

9. Odds

Actual Odds Against Event A

$$\text{actual odds against } A = \frac{P(\overline{A})}{P(A)}.$$

This is usually written as a ratio, such as $a : b$.

Actual Odds in Favor of Event A

$$\text{actual odds in favor of } A = \frac{P(A)}{P(\overline{A})}.$$

If the odds against an event are $a : b$, then the odds in favor are $b : a$.

Payoff Odds

$$\text{payoff odds} = \frac{\text{net profit}}{\text{amount bet}}.$$

Payoff odds are determined by the payout rule, not necessarily by the true probability.

Example 5: If you bet $\$5$ on the number 13 in roulette, your probability of winning is $1/38$. The casino payoff odds are $35 : 1$.

(a) Find the actual odds against winning.

(b) Find the net profit if you win with the casino payoff odds.

(c) If the payoff odds matched the actual odds, what would the net profit be?

Solution

(a) Since

$$P(\text{win}) = \frac{1}{38}, \qquad P(\text{lose}) = \frac{37}{38},$$

the actual odds against winning are

$$\frac{37/38}{1/38} = 37 : 1.$$

(b) The casino payoff odds are $35 : 1$, so a $\$5$ bet gives a net profit of

$$35(5) = 175.$$

The net profit is $\$175$.

(c) If the payoff odds matched the actual odds $37 : 1$, the net profit would be

$$37(5) = 185.$$

The net profit would be $\$185$.

Summary

Key Ideas

- An event is a collection of outcomes.
- A simple event cannot be broken down further.
- The sample space lists all possible simple events.
- Probability values are always between 0 and 1.
- Probability can be found using relative frequency, classical probability, subjective probability, or simulation.
- The complement rule is $P(\overline{A}) = 1 - P(A)$.
- Very small probabilities can identify significant or unusual results.
- Actual odds come from true probabilities; payoff odds come from the payout rule.