Chapter 6 Normal Probability Distributions
Real Applications of Normal Distributions (6.2)

Objectives

In this lesson, we will learn how to

- work with normal distributions that are not standard;
- convert an x value to a z score;
- find probabilities involving real-world normal distributions;
- find an x value when a probability or percentile is given;
- interpret results in context.

1 Nonstandard Normal Distributions

In Section 6.1, we worked with the standard normal distribution, which has

$$\mu = 0, \qquad \sigma = 1.$$

In real applications, a normal distribution often has a mean different from 0 or a standard deviation different from 1.

Key Idea

If a variable x has a normal distribution with mean μ and standard deviation σ, we can convert any x value to a standard normal z score.

2 Conversion Formula

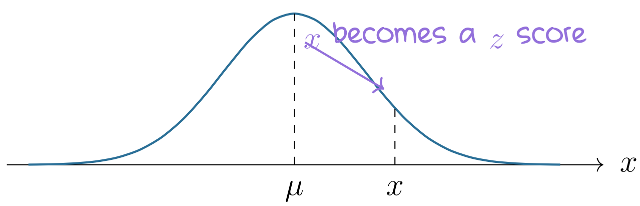
Standardizing an x Value

$$z = \frac{x - \mu}{\sigma}$$

where

- x is the original data value;
- μ is the population mean;
- σ is the population standard deviation;
- z is the number of standard deviations that x is above or below the mean.

Round z scores to two decimal places, unless otherwise instructed.



3 Finding Areas with a Nonstandard Normal Distribution

Procedure

To find a probability involving a nonstandard normal distribution:

1. Sketch a normal curve, label the mean and the given x value(s), and shade the desired region.

2. Convert each boundary value x to a z score using

$$z = \frac{x-\mu}{\sigma}.$$

3. Use technology or Table A-2 to find the area under the standard normal curve.

4. Interpret the probability in the context of the problem.

Example 1: Showerhead Height: Heights of men are normally distributed with mean 68.6 in. and standard deviation 2.8 in.
Find the percentage of men who are taller than a showerhead at 72 in.

Solution

We are given

$$\mu = 68.6, \qquad \sigma = 2.8, \qquad x = 72.$$

The question asks for

$$P(x > 72).$$

First convert $x = 72$ to a z score:

$$z = \frac{x-\mu}{\sigma} = \frac{72-68.6}{2.8} = \frac{3.4}{2.8} \approx 1.21.$$

So we need

$$P(z > 1.21).$$

From technology,

$$P(z > 1.21) \approx 0.1123.$$

Using Table A-2,

$$P(z < 1.21) = 0.8869,$$

so

$$P(z > 1.21) = 1 - 0.8869 = 0.1131.$$

Using technology, the percentage is approximately

$$0.1123 = 11.23\%.$$

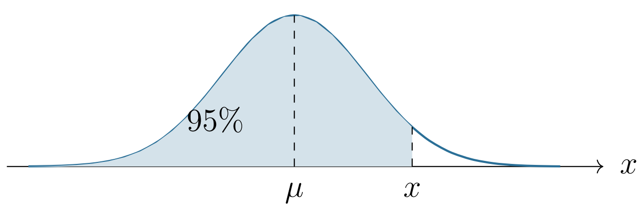
Therefore, about 11.23% of men are taller than 72 inches.

4 Finding Values from Known Areas

Sometimes the probability or percentage is given, and we need to find the corresponding value of x.

Important Reminders

- Do not confuse z scores and areas.
- A z score is a location on the horizontal axis.
- An area is a region under the normal curve.
- A z score on the left side of the mean is negative.
- Areas are always between 0 and 1.



Solving for x

Starting with

$$z = \frac{x - \mu}{\sigma},$$

we can solve for x:

$$x = \mu + z\sigma.$$

This form is useful when the area is known and the original value x is needed.

5 Procedure for Finding x from a Known Area

Procedure

1. Sketch a normal curve and shade the given area.
2. Find the corresponding z score using technology or Table A-2.
3. Convert z back to x using

$$x = \mu + z\sigma.$$

4. Check that the answer makes sense on the graph and in the context of the problem.

Example 2: Designing a Front Door: Assume adult heights are normally distributed with mean 66.2 in. and standard deviation 3.8 in. What door height would allow the shortest 95% of adults to walk through without bending or hitting their heads?
How does this compare with a door height of 80 in.?

Solution

We are given

$$\mu = 66.2, \qquad \sigma = 3.8.$$

We want the value of x with area 0.9500 to the left.
From technology or Table A-2,

$$z \approx 1.645.$$

Now convert the z score back to an x value:

$$x = \mu + z\sigma.$$

Substitute the values:

$$x = 66.2 + (1.645)(3.8).$$

$$x = 66.2 + 6.251 = 72.451.$$

Rounded to one decimal place,

$$x \approx 72.5 \text{ in.}$$

Therefore, a door height of about 72.5 inches would accommodate 95% of adults.
The required door height of 80 in. is much higher than 72.5 in., so it should accommodate more than 95% of adults.

6 Technology Shortcut

Many calculators and software tools can find normal probabilities directly without converting x to z first.

Using Technology

When using technology, you usually enter

$$\mu, \qquad \sigma, \qquad \text{and the boundary value(s)}.$$

Technology can often compute the desired area directly.

Even when technology is used, it is still important to understand the standardizing idea:

$$\text{Nonstandard normal distribution} \quad \longrightarrow \quad \text{Standard normal distribution}.$$

Example 3: Practice: Suppose test scores are normally distributed with mean 75 and standard deviation 8.

(a) Find the probability that a randomly selected score is above 90.

(b) Find the score that separates the bottom 10% from the top 90%.

Solution

(a) We are given

$$\mu = 75, \qquad \sigma = 8, \qquad x = 90.$$

Convert to a z score:

$$z = \frac{90 - 75}{8} = \frac{15}{8} = 1.875 \approx 1.88.$$

So we need

$$P(z > 1.88).$$

Using technology or Table A-2,

$$P(z > 1.88) \approx 0.0301.$$

So about 3.01% of scores are above 90.
(b) The bottom 10% corresponds to a left-tail area of 0.10. Using technology or Table A-2,

$$z \approx -1.28.$$

Now convert back to x:

$$x = \mu + z\sigma = 75 + (-1.28)(8) = 75 - 10.24 = 64.76.$$

So the cutoff score is approximately

$$64.8.$$

Summary

Key Ideas

- A nonstandard normal distribution can have any mean μ and standard deviation σ.
- Convert an x value to a z score using

$$z = \frac{x - \mu}{\sigma}.$$

- Use areas under the normal curve to represent probabilities.
- When an area is known and x is needed, first find z, then use

$$x = \mu + z\sigma.$$

- Always draw or imagine the curve before calculating.