Module 8 Lesson 4
Testing a Claim About a Standard Deviation or Variance (8.4)

Objectives

In this lesson, we will learn how to

- test a claim about a population standard deviation σ;
- test a claim about a population variance σ^2;
- use the chi-square distribution for hypothesis testing;
- write hypotheses involving σ or σ^2;
- use the P-value method, critical value method, and confidence interval method.

1. Key Idea

In this section, we test claims about the variation in a population.

Variation may be described using either the population standard deviation σ or the population variance σ^2.

Main Distribution Used

When testing a claim about σ or σ^2, we use the chi-square distribution, written as

$$\chi^2.$$

2. Notation

Notation

n	sample size
s	sample standard deviation
σ	population standard deviation
s^2	sample variance
σ^2	population variance

3. Requirements

Requirements for Testing Claims About σ or σ^2

1. The sample is a simple random sample.
2. The population has a normal distribution.

Important: The normality requirement is stricter here than it is for many tests about means. Large departures from normality can cause serious errors.

4. Test Statistic

Test Statistic

For a claim about σ or σ^2, use

$$\chi^2 = \frac{(n-1)s^2}{\sigma^2}.$$

Round the test statistic to three decimal places.

The degrees of freedom are

$$df = n - 1.$$

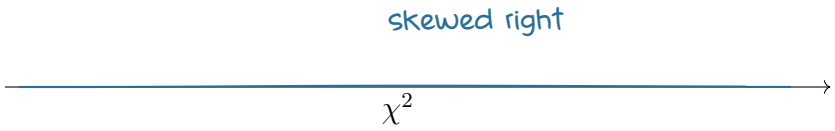
P-values and Critical Values

- P-values may be found using technology or Table A-4.
- Critical values are found using Table A-4 with $df = n - 1$.

5. The Chi-Square Distribution

Properties of the Chi-Square Distribution

- All values of χ^2 are nonnegative.
- The chi-square distribution is not symmetric; it is skewed to the right.
- There is a different chi-square distribution for each number of degrees of freedom.
- In Table A-4, the areas across the top row are areas to the right of the critical value.



6. Equivalent Methods

Equivalent Methods

For tests about σ or σ^2, the following methods are equivalent and will lead to the same conclusion:

- P-value method;
- critical value method;
- confidence interval method.

7. Example: Minting Quarters

Example 1: U.S. Mint specifications require that quarters manufactured after 1964 have weights with a mean of 5.670 g and a standard deviation of 0.062 g.
A new minting process is designed to reduce the standard deviation so that quarter weights are more consistent. The following weights, in grams, are from a simple random sample of quarters:

5.7424	5.7328	5.7268	5.5938	5.6342	5.6839	5.6651	5.6925
5.6803	5.6245	5.7985	5.7180	5.7299	5.6582	5.7360	5.6546
5.7222	5.6619	5.7041	5.6528	5.6210	5.6613	5.6484	5.6502

Use a 0.05 significance level to test the claim that the population standard deviation is less than 0.062 g.

Solution

Requirement check:

1. The sample can be treated as a simple random sample.
2. The sample has no obvious outliers, and a normal quantile plot would show points reasonably close to a straight-line pattern. It is reasonable to proceed with the chi-square test.

Step 1: State the claim.

$$\sigma < 0.062$$

Step 2: Write the opposite of the claim.

$$\sigma \geq 0.062$$

Step 3: Write the hypotheses. Because the claim does not contain equality, it becomes the alternative hypothesis.

$$H_0 : \sigma = 0.062$$

$$H_1 : \sigma < 0.062$$

This is a left-tailed test.

Step 4: Significance level.

$$\alpha = 0.05$$

Step 5: Use the chi-square distribution. The claim is about σ, so we use the χ^2 distribution.

8. Continuing the Quarter Example: P-Value Method

Solution

For the sample,

$$n = 24, \qquad s = 0.0480164.$$

Using the test statistic formula,

$$\chi^2 = \frac{(n-1)s^2}{\sigma^2}$$

we get

$$\chi^2 = \frac{(24-1)(0.0480164)^2}{(0.062)^2}.$$

$$\chi^2 \approx 13.795.$$

Technology gives

$$P\text{-value} \approx 0.0674.$$

Since

$$0.0674 > 0.05,$$

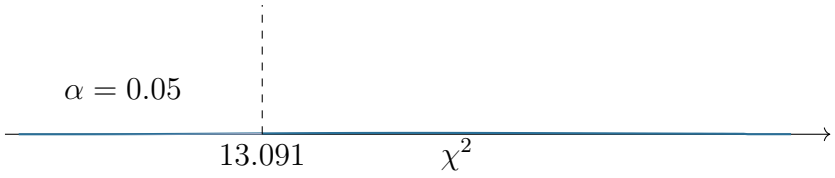
we fail to reject H_0.

Conclusion: There is not sufficient evidence to support the claim that the population standard deviation is less than 0.062 g. In context, there is not enough evidence to show that the new minting process reduced the variation in quarter weights.

9. Critical Value Method

Critical Region for This Example

Because this is a left-tailed test with $\alpha = 0.05$, the critical region is in the left tail of the chi-square distribution.



Example 2: Use the critical value method for the same quarter-weight test.

Solution

For the quarter-weight example,

$$n = 24, \qquad df = n - 1 = 23.$$

For a left-tailed test with $\alpha = 0.05$, Table A-4 gives

$$\chi^2 = 13.091.$$

The test statistic is

$$\chi^2 = 13.795.$$

Because

$$13.795 > 13.091,$$

the test statistic is not in the critical region.

Therefore, we fail to reject H_0.

Conclusion: There is not sufficient evidence to support the claim that the new minting process reduced the population standard deviation below 0.062 g.

10. Confidence Interval Method

For tests about σ or σ^2, the confidence interval method leads to the same conclusion as the P-value and critical value methods.

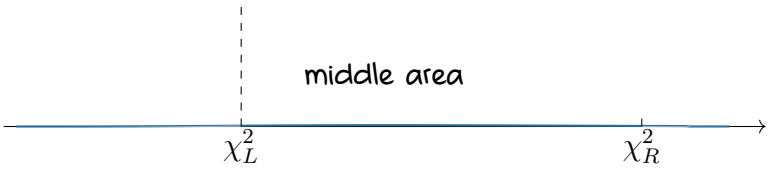
Choosing the Confidence Level

For a one-tailed hypothesis test with significance level α, use a confidence level of

$$1 - 2\alpha.$$

For this left-tailed test with $\alpha = 0.05$, use a

$$90\% \text{ confidence interval.}$$



Example 3: Use a suitable confidence interval to repeat the hypothesis test from the quarter-weight example.

Solution

For the sample,

$$n = 24, \qquad s = 0.0480164, \qquad df = 23.$$

For a 90% confidence interval, the critical values are

$$\chi^2_L = 13.091, \qquad \chi^2_R = 35.172.$$

The confidence interval for σ^2 is

$$\frac{(n-1)s^2}{\chi^2_R} < \sigma^2 < \frac{(n-1)s^2}{\chi^2_L}.$$

To get the confidence interval for σ, take square roots:

$$\sqrt{\frac{(n-1)s^2}{\chi^2_R}} < \sigma < \sqrt{\frac{(n-1)s^2}{\chi^2_L}}.$$

Substituting the values gives

$$0.03883 < \sigma < 0.06365.$$

The claimed value 0.062 g is inside this interval, so the confidence interval does not support the claim that

$$\sigma < 0.062.$$

Conclusion: We fail to reject H_0. There is not sufficient evidence to show that the new minting process reduced the standard deviation of quarter weights.

11. Important Interpretation

What We Can and Cannot Say

Failing to reject H_0 does not prove that

$$\sigma = 0.062.$$

It only means that the sample evidence is not strong enough to support the claim that

$$\sigma < 0.062.$$

12. Summary

Key Ideas

- Claims about σ or σ^2 are tested with the chi-square distribution.
- The test statistic is

$$\chi^2 = \frac{(n-1)s^2}{\sigma^2}.$$

- The degrees of freedom are

$$df = n - 1.$$

- The population must be normally distributed.
- The chi-square distribution is skewed right and is not symmetric.
- For this topic, the P-value method, critical value method, and confidence interval method are equivalent.
- Table A-4 uses areas to the right of the chi-square critical value.